Kinematics:

Kinematics related to motion of objects.

- **Distance & Displacement:**

  - **Distance:** The sum of all side without considering the directions effect.

  - **Displacement:** The shortest line between the final point and the initial point

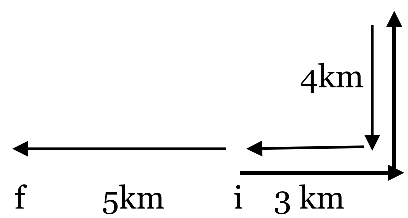
$$\Delta x = x_f - x_i$$

Hint: to find displacement use finger to point on initial and final points and check the direction of finger.

Note: (1) East, north positive and south, west negative.
(2) Distance is scalar and displacement is vector.

Example (1): A man moved 3Km east then 4Km north, then went back to his starting point using the same path and moved 5Km to the west, find distance and displacement:

Solution:



the graph shows the movement of the man form (i) initial point to (f) final point.

a) Distance:

Distance=3+4+4+3+5 km= 19 km

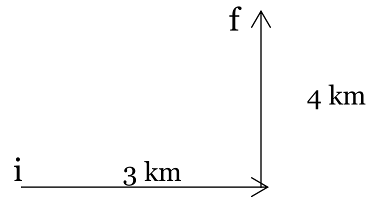
b) Displacement: is the shortest line:

Displacement= $\Delta x = x_f - x_i = -5 - 0 = -5\ km$

The minus sign is for showing the vector; it means that he is moving to the west, you can write as 5 km to west or 5 km ⟵ and show the direction.

Answer Distance 19 km and displacement is 5 km west.

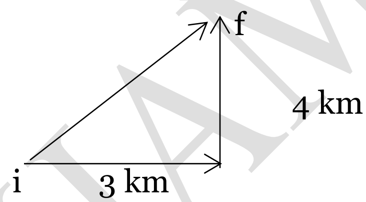
Example (2): A man starts moving from 3 km east then 4 km north to arrive at his destination, find a) distance, b) displacement?
Solution:
Sketch the trip as:



a) Distance: no need for direction

Distance=3+4 km= 7 km

b) Displacement: from initial point to final point the shortest line is as follow:



As seen the shortest line is the hypotenuse using **Pythagorean Theorem**:

$$\sqrt{(3^2+4^2)} = 5\ km \text{ Northeast } \nearrow$$

- **Speed & Velocity:**

$$\textbf{Speed} = \frac{\text{Distance}}{\text{time}}$$

$$\textbf{Velocity} = \frac{\text{dispalcment}}{\text{time}}$$

Distance is scalar and velocity is vector both Units of speed & Velocity is (m/s).

Example (3): From example (2) if the man took time 2 minutes for 3 km and 5 minutes for 4km, find: a) speed, b) displacement:

a) **Speed:**

$$\frac{\text{distance}}{\text{total time}} = \frac{4+3}{2+5} = \frac{7}{7} = 1\ km/min$$

b) **Velocity:**

$$\frac{\text{displacement}}{\text{total time}} = \frac{5}{2+5} = \frac{5}{7} = 0.714\ \text{km/ min}\quad northeast \nearrow$$

Note: Don't forget the direction in the velocity because it is a vector quantity.

- **Acceleration:**

Acceleration is the rate of change of the velocity per unit time.

$$a = \frac{\Delta V}{time} = \frac{V_f - V_i}{time}\quad (\text{m}/s^2)$$

Example (4): A car starts moving from rest and accelerates to a speed of 25 m/s in 5 seconds, find the average acceleration of the car?

Solution:

firstly, write the given data:

V_i = Zero (starts from rest), $V_f = 25\ m/s$, t = 5s, a = ??

$$a = \frac{\Delta V}{time} = \frac{V_f - V_i}{time} = \frac{25-0}{5} = \frac{25}{5} = 5\ (\frac{m}{s^2})$$

When the acceleration is positive means the object is speeding up, and once it is negative it is slowing down.

- **Relation of Velocity & Acceleration:**

	Postitve (+)	Negative (-)	Zero	Constant
Velocity	East or north	South or west	Rest, stop or stationary	$V_f = V_i$ $a = 0$
Acceleration	Speed up	Slow down	$V_f = V_i$ $a = 0$	$V_f \neq V_i$ $a \neq 0$

As seen in the table above, when the **velocity** is **negative** that doesn't mean the object is slowing down but the direction of the motion is in the opposite direction (reverse)

The acceleration is constant means the rate of changing the velocity is constant.

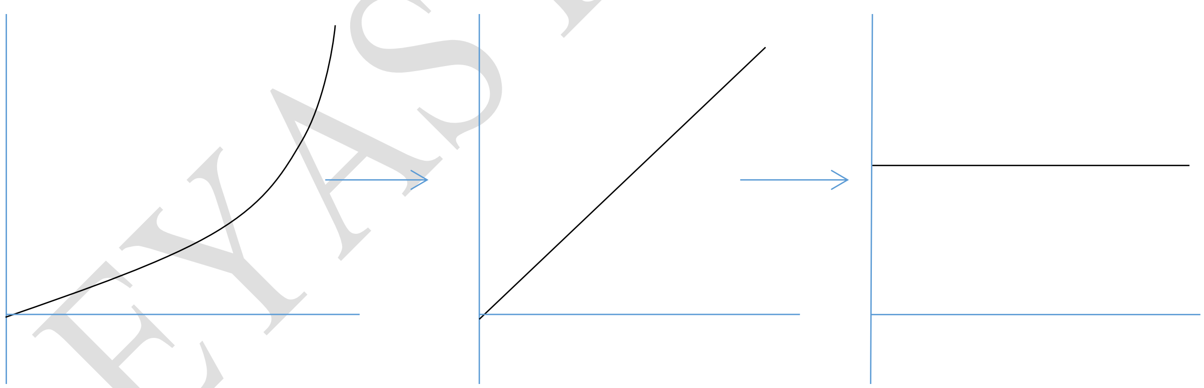
- **Graphing displacement, velocity and acceleration**:

The slope of displacement gives velocity, and the slope of velocity gives the acceleration.

Displacement ⟶ Velocity ⟶ acceleration

As taken before, the graph are arranged as:

Parabola ⟶ Linear ⟶ Constant.



To predict the graph of displacement, velocity and acceleration, use slope and area concept of the graphs.

Assume the acceleration of the object is constant, so the velocity will be linear and the displacement is parabola.

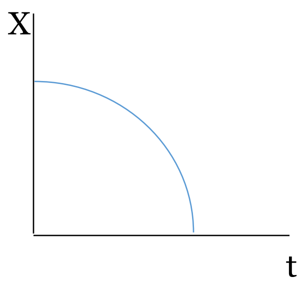
As taking the area under the graph of the given data which is here the acceleration is constant.

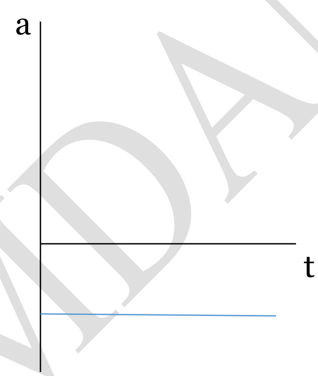
Example (5): What is the graph of displacement vs time for an object that has constant opposite acceleration?

Solution:

From the question, the given data is that the acceleration is constant and opposite (negative). Therefore; to find the graph of displacement from the acceleration, two backward steps must be taken, and the graph of constant will be parabola.

The graph of displacement is negative parabola because the acceleration negative constant.





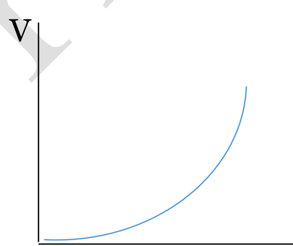
Answer the displacement graph as the left graph shown above.

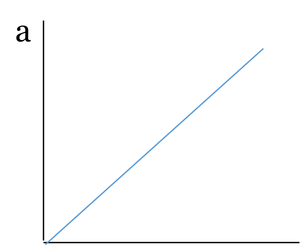
Example (6): What is the graph of acceleration vs time for an object has parabolic increasing velocity vs time?

Solution:

The given velocity is parabola and the acceleration is required, one step forward must be done. Therefore; the parabola will be linear as one step forward.

The graph of acceleration vs time is positive linear.





Answer the right figure is the acceleration vs time.

- **Kinematics Equations:**

1- $V_f = V_i + at$
2- $(V_f)^2 = (V_i)^2 + 2a(\Delta x)$
3- $X_F = X_i + V_i t + \frac{1}{2}at^2$
4- $\Delta X = \frac{V_i + V_f}{2} . t$

Where:
Δx: Displacement $\Delta x = x_f - x_i$
a: Acceleration
V: Velocity

Example (7): A car accelerates from rest at a rate of 5 m/s^2 for a time of l5 s. What is the speed of the car at the end of 15 s?
Solution:

Given V_i = Zero (starts from rest), V_f =?? m/s , t = 15 s, a =5 m/s^2

Using kinematic equation number (1):

$$V_f = V_i + at, \qquad V_f = 0 + (5 \times 15) = 45\ m/s$$

Answer is 45 m/s.

Example (8): A man 10 meter from a goal line, he started running from rest and accelerates at 5 m/s^2, how far is he from the goal line after 10 seconds?

Solution:

given V_i = 0 m/s, $X_i = 10m$, t = 10s, a = 5 m/s^2, X_f =?? m

Using the third kinematic equation:

$$X_F = X_i + V_i t + \frac{1}{2}at^2$$

$$X_F = 10 + (0 \times 10) + \frac{1}{2}(5 \times 10^2) = 260m$$

Answer 260m from goal line.

- **Free fall:**

If the object is released form a height, it will start falling down under the influence of the gravity.

While the object is falling down its velocity increases uniformly until it strikes the ground, this due to the gravitational acceleration which changes the speed of falling objects.

Must know:

$\Delta x = h,$ $\quad a = g = 10\ m/s^2$, maximum height: $V_{final} = zero\ (0)$

If the object is moving upward, it will start to slow down; therefore, gravity is -10.
If it goes down the speed will increase and the gravity is +10.

Example (9): A stone is dropped from rest. What is the acceleration of the stone immediately just after it is released?

Solution:

In free fall situation the acceleration that effects in any object is g (gravitational acceleration) because it is going downward so the answer is g= +10 m/s^2

Example (10): A rock is thrown upward with an initial velocity of 20 m/s find:

a) The acceleration just after it is thrown:
Solution:

The acceleration is always g =10 m/s^2 , because the example said it is thrown upward so the $\mathbf{g = -10\ m/s^2}$

b) The time required to reach maximum height:
Solution**:**

maximum height gives hint to know that the final velocity of the object is **zero** , because when it reach the maximum height the object will stop to change the direction so the **final velocity is zero ,** $\boldsymbol{V_f = 0\ m/s\ (zero)}$

Using the first equation of kinematics:
$V_f = V_i + gt$, take g = -10 (because upward), by substituting

0 = 20 + (-10) t, then **t= 2 s**

c) What is the maximum height that rock can reach?
Solution:

Use second or third kinematics equation to find the height

$$(V_f)^2 = (V_i)^2 + 2a(\Delta x),\ \ (0)^2 = (20)^2 + 2(-10)(\Delta x)$$

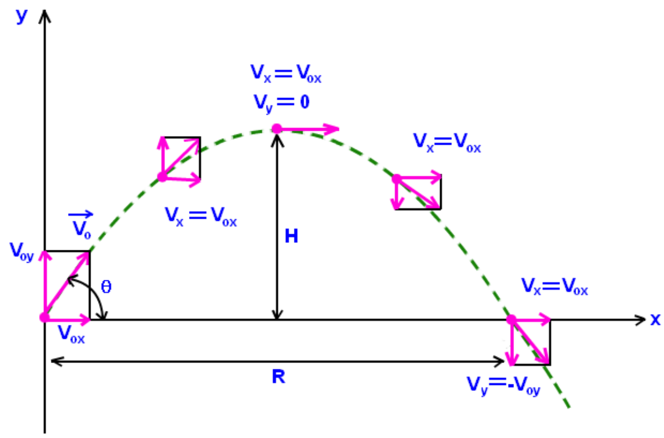
Then:

$$\Delta x = 20\ m\ \ X_F = 20m$$
$$\text{Because } X_i = 0\ m$$

The other method using the third equation:

$$X_F = X_i + V_i t + \frac{1}{2}at^2,$$
$$X_F = 0 + (20 \times 2) + \left(\frac{1}{2} \times -10 \times 2^2\right) = 40 - 20 = \mathbf{20m}$$

- **Projectile motion:**



Projectile motion is the description of an action combined between two different axis in two components x & y component.

From the concept of projectile, to solve the related questions there are simple procedures to follow:

1) Split the initial velocity into two components x & y component by giving sin and cos of angle.

   Y-component V_y , x-component V_x.

2) Take the acceleration for the y-component as a_y=g and for the x-component a_x=0 ($V_{ix} = V_{fx}$)

3) Total time for the trip $t_{total} = t_x = t_y = 2t_{max.height}$

As shown in the above figure, R is the range sets on x-axis and H (height) is on y-axis.

Main equations to use for projectile motion is:
For x-axis:

$$R = V_{ix} \times t_{total}$$

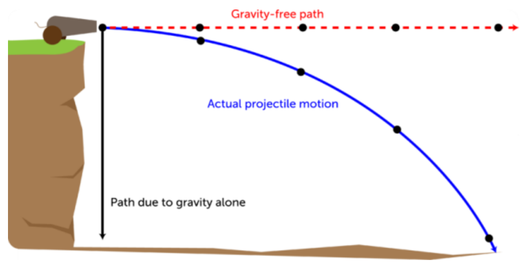
And the kinematic equation on the y-axis:

$$V_{fy} = V_{iy} + gt$$

$$(V_{fy})^2 = (V_{iy})^2 + 2gt$$

$$h = V_{iy}.t + \frac{1}{2}.g.t^2$$

Example (11): A projecitle fired from a canon luanched horizantaly from the edge of cliff 20m high with an initial speed of 10 m/s , find the range that the projectile travles before strikes the ground , see the figure below ?



Solution:

The question mention the velocity to be horizontally. Therefore, the velocity on y-axis is zero.

$$V_{iy} = 0\, m/s \;, \qquad V_{xi} = 10\, m/s.$$

Firstly find the time required to reach the ground:

$$h = V_{yi}t + \frac{1}{2}gt^2,$$

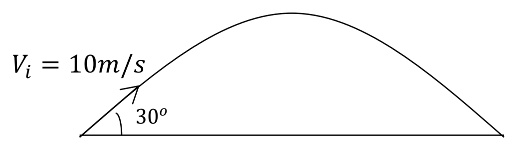
$$h = 0 + (\frac{1}{2} \times 10 \times t^2,$$

$$t_y = \sqrt{\frac{2h}{10}} = 2s,$$

$$t_x = 2s,$$

$$R = V_{xi} \times t_x = 10 \times 2 = \mathbf{20\ m}$$

Example (12): An object is projected at 30^o from the ground at an initial speed of 10m/s



Find:

a) The time required to reach maximum height?

Solution:

$$V_{si} = V_i \times cos(\theta) = 10\ cos(30) = 10 \times 0.87 = \mathbf{8.7\ m/s} \longrightarrow$$

$$V_{yi} = V_i \times sin(\theta) = 10\ sin(30) = 10 \times 0.5 = \mathbf{5\ m/s}\ \uparrow$$

To find the time:

$$V_{yf} = V_{yi} + gt$$

take g = -10 (because upward), by substituting

$V_{yf} = zero\ (\frac{m}{s})$, 0 = 5 + (-10) t, then **t= 0.5 s**

b) The range that the object reached?

Solution:
Total time = 2 × $t_{max.hieght} = 2 \times 0.5 = 1s$, $V_{xi} = V_{xf} = 8.7\ m/s$

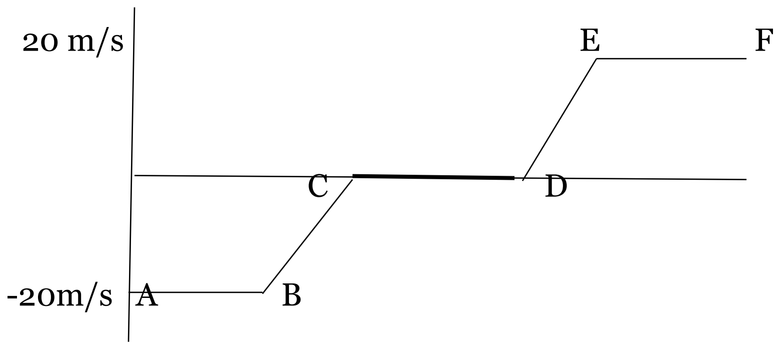
R (range) = $V_{xf} \times t_{total}$ = 8.7 × 1 = $\mathbf{8.7\ m}$

c) The final velocity of the object just before it strikes the ground?

Solution**:**

The final velocity equals the vector of the initial velocity because it starts and falls at the same level of the ground **10 m/s**.

Example (13): If a car is moving from A to F following the figure below, solve the following questions:



a) Find the interval where the acceleration is opposite to the motion of the object?

Solution**:**

acceleration is opposite to the motion means (a) is negative deceleration motion, the answer is from B to C (B-C) , the speed was -20 m/s minus means moving to the west and opposite of it is east so from B to C the car will slow down

Answer is (B-C)

b) The interval where the car is at rest?

Solution:

(C-D) the velocity is zero no motion.

c) The interval where the car is accelerating?

Solution:

(D-E) the car at D has zero velocity and then it started to speed up to reach the 20 m/s at point E.

d) The interval where there is no acceleration?

Solution:

(A-B) & (C-D) & (E-F) at these intervals the velocity remains constant so no acceleration or deceleration.